

has elapsed. Wherever there is an appreciable amount of overdetermination, the use of the least-squares technique as employed here is also recommended.

(6) In the longer wavelength region ( $\lambda > 1.5 \text{ \AA}$ ) the single-crystal spectrometer described here is capable of an accuracy substantially equal to that of a double-crystal instrument and has some advantages from the standpoint of convenience.

*Note added in proof.* The authors have just completed a report giving tables of recommended values for all x-ray emission lines and absorption edges, which is now in the course of publication. In this report, for reasons stated there, the  $W K\alpha_1$  line has been adopted as the *primary standard for x-ray wavelengths*, with the value  $\lambda_{W K\alpha_1} = 0.2090100$ . (This is believed to represent the value in *angstroms* within a probable error of 5 ppm.)

The ratios given in Table VIII are, of course, equally as valid for this definitive choice as for the provisional one used in the present paper.

#### ACKNOWLEDGMENTS

The authors appreciate the assistance of Dr. Ivars Henins and Dr. Jon J. Spijkerman in calibration of the circle and construction of the high-voltage supply. They are further indebted to Dr. Henins for prepublication data on the  $Cu K\alpha_1$  radiation with silicon crystal *S1*. They also wish to thank Lawrence Kurtz for recording much of the data and constructing parts of the electronics, and Alexander F. Burr for manuscript suggestions. The curve shown in Fig. 1 was recorded by Dr. Alan J. Bearden.

## Thermal Boundary Resistance between Solids and Helium below $1^\circ\text{K}$ †

A. C. ANDERSON, J. I. CONNOLLY, AND J. C. WHEATLEY

*Department of Physics and Materials Research Laboratory, University of Illinois, Urbana, Illinois*

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Results are reported from measurements of the thermal boundary resistance between liquid  $\text{He}^3$  and copper or epoxy resin, and between solid  $\text{He}^3$  or liquid  $\text{He}^4$  and copper. Measurements were made in the range 0.05 to  $0.9^\circ\text{K}$  and at pressures up to 395 psi. No quantitative agreement has been found with theory. Above  $0.11^\circ\text{K}$ , the thermal boundary resistance of liquid  $\text{He}^3$  and liquid  $\text{He}^4$  both have a temperature dependence stronger than  $T^{-3}$ . Below  $0.11^\circ\text{K}$  the thermal boundary resistance of  $\text{He}^3$  varies as  $T^{-3}$  over the full pressure range in the liquid. In the case of  $\text{He}^4$  at low pressure, however, the stronger temperature dependence continues to the lowest temperature measured ( $0.075^\circ\text{K}$ ). It is not clear whether the qualitative difference in the behavior of liquid  $\text{He}^3$  and liquid  $\text{He}^4$  is associated with zero sound in liquid  $\text{He}^3$ .

### I. INTRODUCTION

THE transfer of heat between liquid helium and a solid body in contact with the liquid produces a temperature discontinuity at the interface of the two media. This phenomenon, which may be considered as a thermal boundary resistance, was first discovered for liquid  $\text{He}^4$  by Kapitza in 1941.<sup>1</sup> In 1957, Lee and Fairbank<sup>2</sup> observed similar behavior for liquid  $\text{He}^3$ , which served to prove that the boundary resistance was not a characteristic of superfluid  $\text{He}^4$ . Indeed, thermal boundary resistances have been reported for the interface between dissimilar solids at low temperatures,<sup>3,4</sup> including solid  $\text{He}^3$ .<sup>5</sup>

Khalatnikov<sup>6</sup> has developed a theory for the thermal boundary resistance of liquid  $\text{He}^4$  from considerations essentially of the radiation and absorption of thermal phonons at the liquid-solid interface. The boundary resistivity  $R$ , defined as the temperature drop at the boundary divided by the energy flux, may be calculated from the equation

$$R = \frac{15h^3}{16\pi^5 k^4} \frac{\rho_s c_t^3}{\rho_{\text{He}} c_{\text{He}}} \frac{10^{-7}}{F} \frac{1}{T^3}, \quad (1)$$

where  $h$  is Planck's constant,  $k$  Boltzmann's constant,  $\rho_s$  the density of the solid,  $\rho_{\text{He}}$  the density of the helium,  $c_t$  the velocity of transverse acoustic waves in the solid, and  $c_{\text{He}}$  the velocity of acoustic waves in helium.  $F$  is a function of the ratio of velocities of longitudinal and transverse acoustic waves in the solid. Thus the theory predicts a boundary resistance proportional to the inverse third power of the temperature, a dependence on the nature of the solid through  $\rho_s c_t^3/F$ , and a dependence

† Supported in part by the U. S. Atomic Energy Commission.

<sup>1</sup> P. L. Kapitza, *J. Phys. (USSR)* **4**, 181 (1941).

<sup>2</sup> D. M. Lee and H. A. Fairbank, *Phys. Rev.* **116**, 1359 (1959).

<sup>3</sup> A. C. Anderson, G. L. Salinger, and J. C. Wheatley, *Rev. Sci. Instr.* **32**, 1110 (1961).

<sup>4</sup> L. J. Barnes and J. R. Dillinger, *Phys. Rev. Letters* **10**, 287 (1963).

<sup>5</sup> A. C. Anderson, G. L. Salinger, W. A. Steyert, and J. C. Wheatley, *Phys. Rev. Letters* **7**, 295 (1961).

<sup>6</sup> I. M. Khalatnikov, *Zh. Eksperim. i Teor. Fiz.* **22**, 687 (1952).

on the pressure or density of the liquid helium through  $\rho_{\text{He}}c_{\text{He}}$ .

The most recent experimental investigations of the thermal boundary resistance of liquid He<sup>4</sup> have been by Challis,<sup>7</sup> by Johnson and Little,<sup>8</sup> and by Kuang Wey-Yen.<sup>9</sup> These and other authors<sup>10</sup> measure similar magnitudes of boundary resistance, but frequently disagree as to the dependence on temperature, the dependence on surface conditions, or even as to the dependence on the superconducting transition in metals. There is common agreement, however, that the boundary resistance is very sensitive to surface conditions. There is also common *disagreement* with all aspects of the Khalatnikov theory (see note, Ref. 9). The observed boundary resistance, for example, is orders of magnitude smaller than predicted, the measured temperature dependence is usually not  $T^{-3}$ , and there is insufficient sensitivity to the density of the helium or to the material of the solid.

Various theoretical attempts have been made to modify the Khalatnikov theory or to find a parallel mechanism of heat transfer. Challis, Dransfeld, and Wilks<sup>11</sup> have proposed that a dense layer of helium must exist at the interface because of the attractive van der Waals forces which would exist between the atoms of the helium and those of the solid. Such a layer would bring the magnitude and pressure dependence into better agreement with experiment, but the temperature dependence, of order of  $T^{-4}$ , is greater than that usually observed. Little<sup>12</sup> and Andreev<sup>13</sup> have suggested that the electrons of a metallic solid might interact with a surface which is in contact with liquid helium and thus provide, for a metal, a parallel mechanism for the transfer of thermal energy. Thus far such theories have had only limited success in explaining the measured boundary resistance of liquid He<sup>4</sup>.

The theory of Khalatnikov has been extended to the case of low-temperature liquid He<sup>3</sup> by Bekarevich and Khalatnikov.<sup>14</sup> These authors assumed that the Landau Fermi liquid theory<sup>15</sup> would be appropriate to liquid

He<sup>3</sup> below 0.2°K and found that in such case the thermal energy flux from a solid wall would be transmitted primarily into the collective excitations of liquid He<sup>3</sup> known as zero sound. They derived the following expression for the boundary resistivity:

$$R = \frac{5h^3}{8\pi^5 k^4} \frac{\rho_s c_t^3}{\rho_c} \frac{m}{p_0} \frac{10^{-7}}{aF + b\varphi} \frac{1}{T^3}. \quad (2)$$

Here  $m$  is the mass of a He<sup>3</sup> atom and  $p_0$ , the momentum at the surface of the Fermi sphere, is given by

$$N/V = 8\pi p_0^3 / 3h^3, \quad (3)$$

where  $N/V$  is the number of atoms per unit volume.  $\varphi$ , like  $F$ , depends only on the velocities of sound in the solid wall, and  $a$  and  $b$  depend on the conditions of reflection of quasiparticles at the wall and on the properties of the liquid. The boundary resistance again is proportional to the inverse third power of the temperature, and is sensitive to the pressure on the liquid and the material of the wall.

Bekarevich and Khalatnikov hypothesized that at higher temperatures, where the Fermi liquid theory would not be applicable, the heat transfer would again be via ordinary phonons as derived by Khalatnikov. Thus, both at low and at high temperatures, the thermal boundary resistance of liquid He<sup>3</sup> would have a temperature dependence of  $T^{-3}$ , with a relative increase in thermal resistance in going to lower temperatures, the transition occurring near 0.2°K. Thus, a measurement of the thermal boundary resistance of liquid He<sup>3</sup> over a temperature range sufficiently wide to include the transition temperature region would provide a means for detecting the existence of zero sound.

Other methods had been used in an attempt to detect zero sound and had failed. In order for zero sound to propagate, the condition  $\omega\tau \gg 1$  must be fulfilled,<sup>16</sup> where  $\omega$  is the angular frequency of the sound and  $\tau$  is an appropriate mean lifetime of the quasiparticle states (of order  $10^{-12}/T^2 \text{ sec}^\circ\text{K}^2$ ). At frequencies for which  $\omega\tau$  approaches unity, the attenuation becomes very large. For this reason an acoustic pulse-echo experiment carried down to 0.008°K at 25 Mc/sec was not successful.<sup>16</sup> In order to circumvent these problems, experiments involving the scattering of gamma rays or slow neutrons have been proposed.<sup>17,18</sup> Such experiments also involve technical difficulties and thus far an observation of zero sound by such means has not been reported. We were therefore prompted to investigate the thermal boundary resistance.

<sup>16</sup> W. R. Abel, A. C. Anderson, and J. C. Wheatley, Phys. Rev. Letters 7, 299 (1961).

<sup>17</sup> A. A. Abrikosov and I. M. Khalatnikov, Zh. Eksperim. i Teor. Fiz. 41, 544 (1961) [English transl.: Soviet Phys.—JETP 14, 389 (1962)].

<sup>18</sup> A. I. Akhiezer, I. A. Akhiezer, and I. Ya. Pomeranchuk, Zh. Eksperim. i Teor. Fiz. 41, 478 (1961) [English transl.: Soviet Phys.—JETP 14, 343 (1962)].

<sup>7</sup> L. J. Challis, Proc. Phys. Soc. (London) 80, 759 (1962).

<sup>8</sup> R. C. Johnson and W. A. Little, Phys. Rev. 130, 596 (1963).

<sup>9</sup> Kuang Wey-Yen, Zh. Eksperim. i Teor. Fiz. 42, 921 (1962) [English transl.: Soviet Phys.—JETP 15, 635 (1962)]. Note: This author reports a rather close agreement between a portion of his data and the theory of Khalatnikov. It appears to us, however, that the theoretical values with which he compares his data are incorrect by a factor of about 5 or 6.

<sup>10</sup> References 6, 7, and 8 list the numerous earlier works.

<sup>11</sup> L. J. Challis, K. Dransfeld and J. Wilks, Proc. Roy. Soc. (London) A260, 31 (1961).

<sup>12</sup> W. A. Little, Phys. Rev. 123, 435 (1961).

<sup>13</sup> A. F. Andreev, Zh. Eksperim. i Teor. Fiz. 43, 1535 (1962) [English transl.: Soviet Phys.—JETP 16, 1084 (1963)].

<sup>14</sup> I. L. Bekarevich and I. M. Khalatnikov, Zh. Eksperim. i Teor. Fiz. 39, 1699 (1960) [English transl.: Soviet Phys.—JETP 12, 1187 (1961)].

<sup>15</sup> A. A. Abrikosov and I. M. Khalatnikov, Reports on Progress in Physics (The Physical Society, London, 1959), Vol. 22, p. 329. This review lists earlier works on the Landau theory of a Fermi liquid.

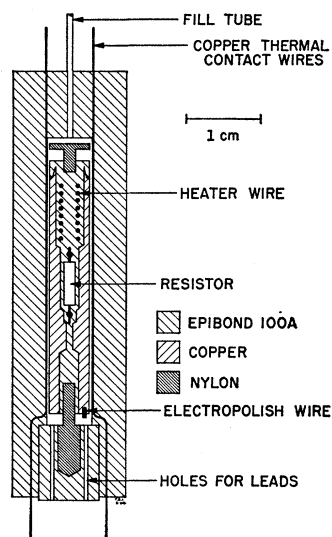


FIG. 1. Cell No. 1. For greater clarity, spaces have been shown between the closely fitted parts and the electrical leads have not been shown. The spacing between the cylinders was about 0.025 in.

We had made previously, in conjunction with other experiments, three measurements of the low temperature ( $0.01^\circ\text{K} < T < 0.05^\circ\text{K}$ ) boundary resistance between liquid  $\text{He}^3$  and epoxy walls.<sup>5,19,20</sup> The boundary resistance did obey a  $T^{-3}$  dependence as predicted by Bekarevich and Khalatnikov. However, a quantitative comparison with theory was not possible since the properties of the epoxy resin were not known. Similarly, a comparison with the high-temperature ( $T > 0.3^\circ\text{K}$ ) data of Lee and Fairbank<sup>2</sup> on a  $\text{He}^3$ -copper interface was not possible.

The reproducible  $T^{-3}$  dependence of our early boundary resistance measurements suggested the feasibility of an investigation over a wider temperature range in an attempt to observe a transition temperature region, in spite of the rather dismal status of the measurements with  $\text{He}^4$  at higher temperatures. Thus we planned to measure the boundary resistance of liquid  $\text{He}^3$  over a temperature range sufficiently great to include the predicted transition region. In addition, for comparative purposes, the pressure dependence would be measured, as well as the boundary resistance of solid  $\text{He}^3$  and liquid  $\text{He}^4$ .

During the course of our work Keen, Mathews, and Wilks<sup>21</sup> published the results of their measurements on the damping of acoustic waves at the ends of a quartz delay line which had been immersed in liquid helium. Their results indicate an abrupt increase in the damping below  $0.09^\circ\text{K}$  (at 1000 Mc/sec) which was shown to be characteristic of liquid  $\text{He}^3$  and may indicate the presence of zero sound. An analysis of their data in

<sup>19</sup> A. C. Anderson, G. L. Salinger, W. A. Steyert, and J. C. Wheatley, *Phys. Rev. Letters* **6**, 331 (1961).

<sup>20</sup> A. C. Anderson, G. L. Salinger, and J. C. Wheatley, *Phys. Rev. Letters* **6**, 443 (1961).

<sup>21</sup> B. E. Keen, P. W. Mathews, and J. Wilks, *Phys. Letters* **5**, 5 (1963).

terms of the Fermi liquid theory had not been published at the time of this writing.

## II. EXPERIMENTAL TECHNIQUE

Two different cells were used in the present measurements on the thermal boundary resistance. These will be described below in detail. In addition, relevant remarks will be made concerning the technique employed in our earlier work below  $0.05^\circ\text{K}$ .

### A. Cell No. 1

A scale drawing of the cell is shown in Fig. 1. The use of a coaxial geometry consisting of an internal copper cylinder containing heater and thermometer and supported by nylon posts of negligible thermal conductance<sup>22</sup> made possible a relatively thick outer wall which could withstand internal pressures of at least 1000 psi. The more commonly used linear geometry requires thin walls. The coaxial design also served to define fairly well the area of contact between helium and solid. In addition, this area of contact remained essentially constant with variations in temperature. There could be, for example, no appreciable heat exchange with the helium via the surface of the nylon supports.<sup>23</sup>

The copper cylinder was constructed from a single piece of annealed, 99.999% pure copper rod. The resistance thermometer was a Speer 220 $\Omega$ ,  $\frac{1}{4}$ -W, grade 1025 carbon resistor<sup>24</sup> with leads of 0.005-in.-diam manganin wire. The heater consisted of a spiral or 0.002-in.-diam Evanohm<sup>25</sup> wire, 12 in. long, with leads of 0.005-in.-diam manganin hard-soldered on. The leads were then coated with soft solder and varnished. The resistor and heater were sealed into a hole in the copper cylinder using Epibond 100-A,<sup>26</sup> a thermal setting epoxy resin. The surface was then electropolished by immersing the cylinder in a stirred bath of orthophosphoric acid (500 to 1000 g/liter  $\text{H}_2\text{O}$ ), the cylinder acting as cathode, and polishing at the rate of 0.40 A for 30 min.<sup>27</sup> The electrical lead to the cylinder and the exposed copper at the end of the cylinder were insulated with GE-7031 varnish. After the polish the surface was quickly and carefully rinsed with distilled water, and the electrical lead clipped off.

The outer wall of the cell consisted of Epibond 100-A with about 400 No. 38 insulated copper wires embedded in the inner surface. These wires were used for thermal contact to the exterior of the cell. The cell

<sup>22</sup> A. C. Anderson, W. Reese, and J. C. Wheatley, *Rev. Sci. Instr.* **34**, 1386 (1963).

<sup>23</sup> J. Jeener and G. Seidel in *Proceedings, of the Seventh International Conference on Low Temperature Physics* (University of Toronto Press, Toronto, 1961, p. 483.)

<sup>24</sup> Speer Carbon Company, Bradford, Pennsylvania.

<sup>25</sup> Wilbur B. Driver Company, Newark, New Jersey.

<sup>26</sup> Furane Plastics, Inc., Los Angeles, California.

<sup>27</sup> W. J. M. Tegart, *The Electrolytic and Chemical Polishing of Metals in Research and Industry* (Pergamon Press, Ltd., London, 1956).

was constructed in the following sequence. A Teflon form of circular cross section was machined to the shape indicated by the copper cooling wires in Fig. 1. The copper wires were laid on the form and held in place by cotton thread while a thin layer of 100-A was sprinkled on and cured. This was set into a "cup" machined from 100-A, the narrow end of the Teflon form plus wires protruding through a hole in the bottom of the cup. The space between wires and cup was filled with molten 100-A and cured. The Teflon form was then partially extracted and the resulting cavities at each end filled with 100-A so as to form a circular cylinder of 100-A with one end closed. The closed end was drilled and the  $\frac{1}{32}$ -in.-o.d. cupronickel tube was sealed in, also using 100-A. The open end was bored to accept the 100-A plug on which the internal copper cylinder had been mounted. This plug was sealed in using Epibond 121,<sup>26</sup> as were the leads which were brought out through the plug.

The copper wires which protruded from both the top and the bottom of the cell were formed into several flat "foils" using GE-7031 varnish. The foils from the top of the cell were placed directly in contact with slabs cut from single crystals of chromium potassium alum.<sup>3</sup> Apiezon N-grease was used as a thermal contact agent. The alum was also in contact with another set of foils which were soldered to a lead thermal switch. The upper end of the thermal switch was connected to a He<sup>3</sup> refrigerator. This switch was activated by the fringing magnetic field of a 12-in. Varian magnet used in the adiabatic demagnetization cycle.

The foils from the bottom of the cell were placed in direct thermal contact with six single crystals of cerium magnesium nitrate (CMN) totaling 0.78 gm. The CMN was used as a magnetic susceptibility thermometer and was calibrated against the vapor pressure of liquid He<sup>4</sup>. The magnetic susceptibility was measured by means of a 17-cps electronic mutual inductance bridge.<sup>28</sup>

Hollow tubes of Teflon were fitted tightly over both ends of the cell. The upper tube contained the alum and served as support for the cell. The lower tube contained the CMN. Both tubes were partially filled with Dow-Corning silicone oil of 1000 centistokes viscosity. This oil solidifies at low temperature and was expected to help prevent vibrational heating of the enclosed components.

Power dissipation in the heater was determined from potential and current measurements made with a Leeds and Northrup type K-3 potentiometer. A 33-cps resistance bridge with phase sensitive detector was used in conjunction with the resistance thermometer. The power dissipation in the resistance thermometer was maintained at an indiscernible level, which at 0.05°K was roughly  $10^{-12}$  W. The pressure of the helium within the cell was determined by a dead weight tester which also

served as a manostat provided that the weights were kept spinning.

It may be noted from the above discussion that one helium-solid interface was of carefully prepared copper while the other interface was between helium and a rather nondescript epoxy surface. This design evolved because it had been originally planned to measure the abrupt increase in temperature of the internal copper cylinder at the instant the heater was turned on. Thus the boundary resistance of the second interface would not be measured and the surface condition would be of little importance. In practice, however, this mode of operation was not very successful because of the relative magnitudes of the various time constants involved in such a measurement.

The measurements to be reported here were made only after the thermal transients, associated with turning the heater on or off, had been dissipated. The resistance thermometer was calibrated by means of the CMN with the heater power off. The heater would then be turned on and the decrease in resistance (increase in temperature) measured. The applied power was such that the fractional change in temperature was less than 10%. A correction was applied, when required, for the slight increase in temperature of the outer wall which resulted from the thermal impedance between the cell and the alum heat sink. The temperature of the outer wall was monitored with the CMN thermometer.

The lowest temperature at which useful data could be obtained, roughly 0.05°K, was determined by the heat leak to the inner copper cylinder. At lower temperatures the heat leak maintained the inner cylinder at a temperature sufficiently greater than that of the outer wall that it was impossible to calibrate the resistance thermometer. One controllable source of heating of the copper cylinder was from eddy currents induced by the 17-cps mutual inductance coils. In order to calibrate the resistance thermometer it was therefore necessary first to measure the temperature of the alum refrigerator by means of the magnetic thermometer, then switch off the 17-cps primary current and measure the resistance of the Speer resistor. The resistor was, thereby, assumed to be at the temperature of the alum.

A more serious defect in the design became important at higher temperatures. The energy dissipated in the heater was absorbed by the alum refrigerator *and* by the He<sup>3</sup> in the cell. Above 0.1°K the heat capacity of the He<sup>3</sup> became appreciable compared to that of the alum refrigerator. Hence the heat flux through the epoxy resin-helium interface was less than that through the copper-helium interface and indeed was not known. In retrospect it was learned that the predominant temperature jump was across the copper-helium boundary and so this problem was not so serious as at first feared. It was primarily this problem which led us to the design of the second cell.

<sup>28</sup> W. R. Abel, A. C. Anderson, and J. C. Wheatley, Rev. Sci. Instr. **35**, 444 (1964).

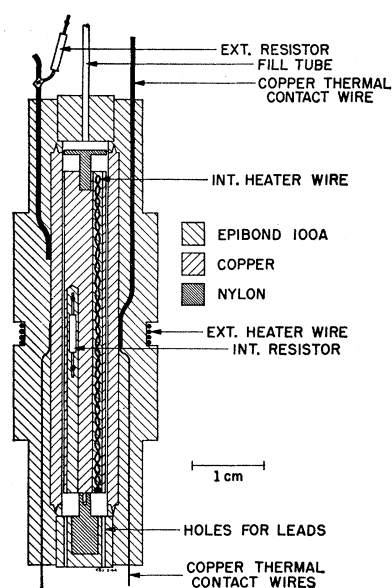


FIG. 2. Cell No. 2. For greater clarity, spaces have been shown between the closely fitted parts and the electrical leads have not been shown. The spacing between the cylinders was 0.0037 in.

### B. Cell No. 2

A scale drawing of this cell is shown in Fig. 2. The cell has two important features. First, the helium exchanges heat only with walls of electropolished, annealed, 99.999% pure copper. Second, a resistance thermometer and heater were thermally coupled to the outer cylindrical copper wall. Power was continuously dissipated in the "external" heater and was adjusted so as to maintain the "external" resistance thermometer at a constant temperature while thermal boundary resistance measurements were being made. Thus the external heater compensated for the gradual warming of the alum refrigerator and for changes in heat flux originating with the heater in the internal copper cylinder. The measurements proceeded at a constant temperature and the problem associated with the heat capacity of the liquid  $\text{He}^3$ , as encountered with cell No. 1, was avoided.

The inner copper cylinder was constructed in a manner similar to that used for cell No. 1, except the heater and resistor (Speer 470  $\Omega$ ,  $\frac{1}{10}$  W, grade 1002) were sealed into separate holes and the heater extended the length of the cylinder so as to provide a more uniform heating. An additional wire was soldered to the inner cylinder and brought out of the completed cell with the other leads so as to permit an electrical continuity check between the two copper surfaces to detect if they touched after the cell was sealed.

The outer copper cylinder was constructed from a solid copper rod with a hole drilled axially through it. Six #22 bare copper wires and 135 #38 insulated copper wires were hard soldered about its midpoint as shown in Fig. 2. A seventh #22 copper wire was soldered at a slightly higher position, as shown. This wire was eventually soldered to the external resistance thermometer which was used to measure the temperature of the outer

copper cylinder. The cylinder was cast into Epibond 100-A, the hole reamed to the required i.d., and the ends bored to accept 100-A plugs. The inner copper surface was then electropolished by placing an anode consisting of a  $\frac{1}{8}$ -in. copper rod coaxially through the cylinder. The orthophosphoric acid solution was forced through the cylinder under the pressure of a 90-cm head of solution. The resulting rate of polishing was 1.2 A for 6 min. After the experiments were finished, the cell was opened and the radial separation of the two copper surfaces measured to be 0.0037 in. on the average.

The #38 wires which protruded from the bottom of the cell were treated in the same manner as described for cell No. 1. The six #22 wires protruding from the top were soft soldered to a loop of #22 wire. Two copper wires, each 0.007 in. diam by 4 cm long, were soldered to this loop. The upper ends of the two wires were soldered to the foils of a chromium potassium alum refrigerator. The pair of wires served as a thermal impedance between the cell and the alum refrigerator so as to allow the external heater to provide a better control over the temperature of the cell.

The mode of operation for this cell was similar to that for cell No. 1, except the outer wall was maintained at a constant temperature during a measurement. With cell No. 2 there was no eddy current heating from the 17-cps magnetic thermometer since there was a greater separation between the cell and the coils. Hence the internal resistor could be calibrated directly against the temperature of the CMN. In measuring the boundary resistance at a given temperature, at least three heater on-off steps were observed. The heater power was varied from as low as 0.16 erg/sec at 0.07°K to 820 erg/sec at 0.84°K, each of which corresponded to a temperature difference of about  $2 \times 10^{-3}$ °K. Occasionally the internal heater power was varied at a fixed temperature so as to check the dependence of the boundary resistance on the magnitude of the heat flux. No such dependence was observed.

After the cell had been sealed and cooled to liquid-helium temperatures, it was discovered that the inner copper cylinder was in electrical contact with the outer copper wall. Therefore, before each run the cell was flushed and pumped at room temperature, then cooled, and the thermal resistance between the two copper cylinders measured. This resistance was reproducible and could be represented by the expression

$$\Delta T/\dot{Q} = 4 \times 10^{-3} T^{-2.3} \text{K}^{3.3} \times \text{sec/erg} \quad (0.2^\circ\text{K} < T < 0.7^\circ\text{K}). \quad (4)$$

This was always at least 500 times larger than the thermal resistance measured with helium in the cell.

With the cell empty and the heater turned off, minimum temperatures reached after demagnetization were 0.08°K for the inner cylinder and about 0.03°K for the outer. Inserting these values into the integrated form of Eq. (4), assuming an extrapolation to lower tempera-

tures is permissible, gives a calculated heat leak to the inner cell of about 1 erg/min. Part of this heat leak is known to have been produced by vibrations caused by a mechanical pump, and part was due to joule heating from rf sources external to our laboratory.

Joule heating of the resistance thermometer by rf fields contributed most of the scatter at the lowest temperatures since a fluctuation in the rf background would in effect shift the calibration of the resistors. This problem was more serious at lower temperatures not only because of the worsening thermal contact between the resistance element and its environment, but also because the thermal time constant of the cell when filled with liquid He<sup>3</sup> became very long (about one hour near 0.08°K). Hence there was a proportionally greater chance of a fluctuation in the rf level during the course of a measurement. Capacitors of 1000 pF value had been soldered across all electrical circuits where they entered the low-temperature region of the cryostat, but these did not reduce the rf heating sufficiently. Fluctuations in the rf level were much less troublesome in measurements on solid He<sup>3</sup> or liquid He<sup>4</sup> because of the smaller heat capacities and, therefore, much shorter time constants.

Temperatures below 0.35°K were obtained by adiabatic demagnetization of the alum refrigerator. For work above 0.35°K the alum refrigerator was removed and the six # 22 wires from the cell were soldered to a ½-in.-diam copper rod which extended from the cell to the He<sup>3</sup> refrigerator. A heater was attached to the He<sup>3</sup> refrigerator which, together with varying the pumping speed, permitted control of the temperature of the refrigerator up to 0.9°K.

The measurements on liquid He<sup>4</sup> were made in a manner identical to that which has been described, except for an additional modification to the cryostat. A small reservoir of 1 cc volume was constructed similar to the outer wall of cell No. 1; that is, with copper wires embedded in the inner surface. The reservoir was placed in series with the ¼-in.-o.d. cupronickel fill-tube from the thermal boundary resistance cell. The copper wires were thermally attached to the He<sup>3</sup> refrigerator. The ¼-in.-o.d. fill-tube was lengthened by inserting a spiral of 50 cm of ¼-in.-o.d. tubing immediately below the reservoir and a second spiral of 90 cm of similar tubing immediately above the reservoir. The purpose of the spirals was to decrease the heat leak down the helium filled tube. The fill-tube diameter was increased to ¼ in. where it entered the 1°K helium bath. The diameter was again increased to ½ in. at room temperature. The ½-in. tube was coupled to a manifold which made provision for filling the cell, reservoir, and fill-tube, and for either pumping or applying a known pressure.

To obtain low temperatures the following procedure was used. The entire system from thermal boundary resistance cell to the fill-tube in the He<sup>4</sup> bath was first filled with liquid He<sup>4</sup> under the conditions that (a) the He<sup>4</sup> bath was pumped to 1°K, (b) the He<sup>3</sup> refrigerator

was operating, and (c) the level of the He<sup>4</sup> bath was low. The fill-tube was then connected to a vacuum pump and the He<sup>4</sup> bath was warmed by increasing the bath pressure to one atmosphere. This procedure boiled off the liquid He<sup>4</sup> within the fill-tube to a level somewhere above the reservoir. Liquid He<sup>4</sup> was then transferred to the bath, the bath was pumped to 1°K, and the adiabatic demagnetization cycle was initiated. The fill-tube was continuously pumped on to prevent reflux of the He<sup>4</sup> vapor from the high-temperature extent of the superfluid helium film.<sup>29</sup> The measurements at high pressure were conducted, of course, with the entire fill-tube at high pressure. Under these conditions the increased heat leak limited our measurements to above 0.7°K.

The He<sup>3</sup> measurements were carried out prior to the measurements on He<sup>4</sup>. The gas used in the He<sup>3</sup> experiments had a He<sup>4</sup> impurity of less than 0.004%. The cell was repeatedly flushed at room temperature with He<sup>4</sup> gas before starting the He<sup>4</sup> measurements.

### C. Earlier Measurements

In our measurements of the heat capacity of liquid He<sup>3</sup> we also had an opportunity to measure the boundary resistance at the interface between liquid He<sup>3</sup> and Epi-bond 100-A surfaces.<sup>5,19</sup> These cells were similar in construction to cell No. 1 of the present work except the interior was packed with powdered CMN only. The CMN was used both as a magnetic susceptibility thermometer and as the second stage in the adiabatic demagnetization cycle. Since the heat capacity of the interior of the cell (He<sup>3</sup> and CMN) was known, the boundary resistance could be determined from the thermal time constant which was measured as the contents of the cell warmed (or cooled) due to the heat leak. The data obeyed the relation

$$\dot{Q} = B(T_{\text{hot}}^4 - T_{\text{cold}}^4) \quad (5)$$

from 0.01°K to about 0.05°K. The effect of applying pressure to the He<sup>3</sup> was also measured.

Although these measurements demonstrated a  $T^{-3}$  dependence of the thermal boundary resistance, they were of only limited value for purposes of comparison with theory. This is because (1) neither the elastic properties nor the proper density of the epoxy resin wall are known as the epoxy contains tiny voids, (2) the effective area of the boundary is unknown since there were many pockets and fissures in the surface, and (3) the measurements included thermal resistances of unknown magnitude, but with a  $T^{-3}$  temperature dependence, which were in series with the He<sup>3</sup>-epoxy resin boundary resistance. The thermal resistance between copper wires and the alum refrigerator would be an example of the latter difficulty.<sup>3</sup> For these reasons neither the magnitude of the thermal resistance nor its

<sup>29</sup> R. P. Hudson, B. Hunt, and N. Kurti, Proc. Phys. Soc. (London) A62, 392 (1949).

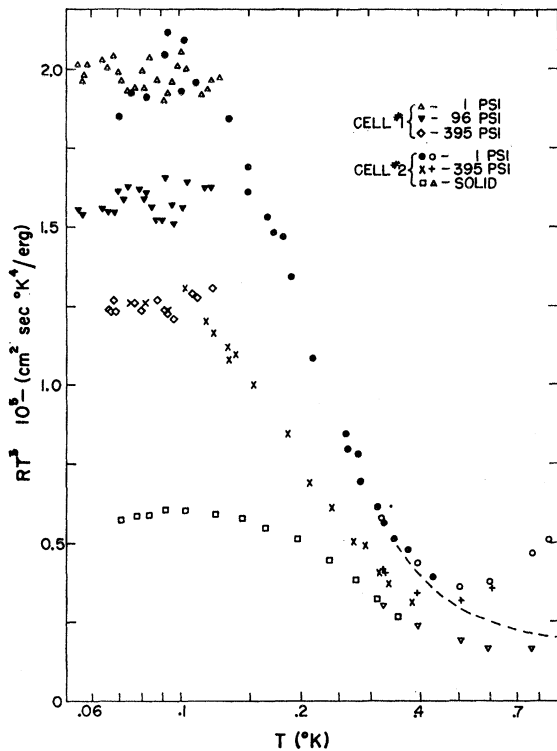


FIG. 3. The product, of the thermal boundary resistivity  $R$  of liquid or solid  $\text{He}^3$  multiplied by the third power of the temperature, as a function of temperature. The high-temperature liquid- $\text{He}^3$  data must be corrected for the bulk thermal resistance of the liquid itself. The corrected data at 1 psi are indicated by the broken line. The low-temperature data were obtained separately from the high-temperature data, as indicated by the different symbols.

pressure dependence can be compared with theory. For technical purposes we give the thermal resistivity between liquid  $\text{He}^3$  and walls of Epibond 100-A containing copper wires as

$$R \approx 3 \times 10^{-6} / T^3 (\text{cm}^2 \text{sec}^4 \text{K}^4 / \text{erg}). \quad (6)$$

Note that this value is not corrected for thermal resistances in series with the boundary resistance, and also is calculated assuming a smooth, cylindrical surface.

A determination of the boundary resistance between liquid  $\text{He}^3$  and Formex insulated copper wires was made in conjunction with a measurement of the thermal conductivity of liquid  $\text{He}^3$ .<sup>20</sup> The thermal contact to the  $\text{He}^3$  was made via a "brush" of 3500 #44 copper wires, each wire extending 0.4 cm into the liquid. It was possible so to analyze the data that one could extract the thermal boundary resistance between just the insulated copper wires and the  $\text{He}^3$ . The boundary resistivity was found to be

$$R = 7 \times 10^{-6} / T^3 (\text{cm}^2 \text{sec}^4 \text{K}^4 / \text{erg}) \times (0.025^\circ \text{K} < T < 0.04^\circ \text{K}). \quad (7)$$

The greatest uncertainty in calculating this value was

in the area of contact ( $27 \text{ cm}^2$ ), which may have been in error by  $\pm 20\%$ . Again, Eq. (7) is given only for technical purposes since the influence of the Formex insulation is not known.

### III. EXPERIMENTAL RESULTS

The thermal boundary resistance data obtained with cell No. 2 are shown in Figs. 3, 4, and 5. The thermal resistivity  $R$  has been multiplied by  $T^3$  in order better to portray the relationship with theory. The effective wall area involved in the heat transfer probably increased slightly at lower temperatures, especially for the measurements on liquid  $\text{He}^3$ , because the ends of cylinders transfer proportionally more heat to the liquid. In the computation of the boundary resistances a value of  $9.27 \pm 0.31 \text{ cm}^2$  has been used for the effective area of each copper cylinder. This is a mean value and the limits indicate the variation which might be expected at low or high temperatures.

The data for temperatures below  $0.35^\circ \text{K}$  were obtained separately for those above  $0.35^\circ \text{K}$  as was explained in the section on experimental technique. It will be noted that there is excellent overlap for the  $\text{He}^3$  runs, but that there is a discrepancy between the low- and high-temperature  $\text{He}^4$  runs. The reason for this discrepancy is not understood.

The thermal resistance measurements for liquid  $\text{He}^3$

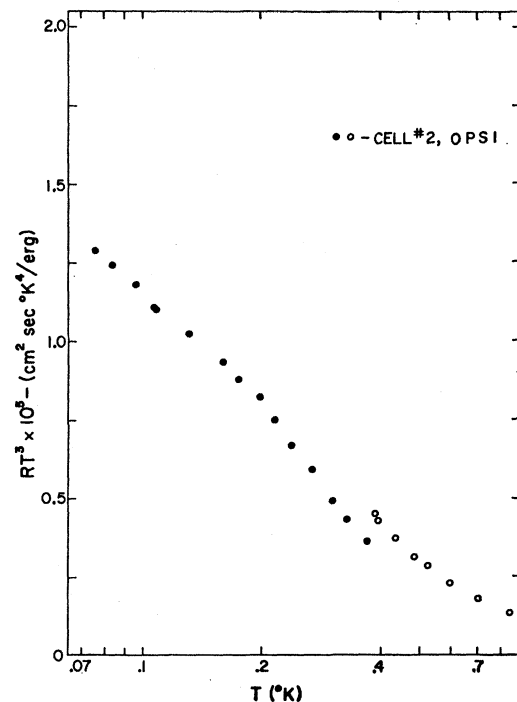


FIG. 4. The product, of the thermal boundary resistivity  $R$  of liquid  $\text{He}^4$  multiplied by the third power of the temperature, as a function of temperature. The coordinates of this figure are identical in scale to those of Fig. 3.

include a contribution from the bulk thermal resistance of a 0.0037-in.-thick layer of liquid He<sup>3</sup>. A correction for this effect becomes significant only above 0.35°K. Since the thermal conductivity of liquid He<sup>3</sup> has been measured only at low pressure,<sup>2</sup> only the 1-psi data can be corrected. The corrected data are indicated by the broken line in Fig. 3. Obviously above 0.5°K the correction for the thermal resistance of the He<sup>3</sup> itself is so large that the final value for  $RT^3$  is not precise.

An investigation of the effect of pressure on the boundary resistance of liquid He<sup>3</sup> and liquid He<sup>4</sup> revealed an irreversible phenomenon which had not been reported previously. These data are shown in Fig. 5. A curve of  $RT^3$  versus  $T$  was established at low pressure (1 psi for He<sup>3</sup> and effectively zero pressure for He<sup>4</sup>) and was reproducible within the indicated scatter. The pressure was next increased at a fixed temperature to 395 psi in the case of He<sup>3</sup> or 300 psi in the case of He<sup>4</sup> and a data point taken. The pressure was then reduced to its original low value. The change in thermal resistance during the initial increase in pressure was larger than the change observed with a reduction in pressure. Henceforth the curve of  $RT^3$  versus  $T$  at 395 psi and the new curve for 1 psi were reproducible and independent of further cycling of the pressure between 1 and 395 psi. For liquid He<sup>3</sup> the irreversibility could not be distinguished at temperatures less than 0.3°K. There was

TABLE I. Ratio of the thermal boundary resistance  $R_1$  of liquid He<sup>4</sup> at zero pressure or liquid He<sup>3</sup> at a pressure of 1 psi to the boundary resistance  $R_p$  at higher pressures. The He<sup>3</sup> ratios have not been corrected for the bulk thermal resistance of the liquid.

Cell No.	Liquid	$T(^{\circ}K)$	$P$ (psi)	$R_1/R_p$
1	He <sup>3</sup>	<0.1	96	1.25
1	He <sup>3</sup>	<0.1	395	1.58
2	He <sup>3</sup>	<0.1	395	1.58
2	He <sup>3</sup>	0.323	395	1.41
2	He <sup>3</sup>	0.396	395	1.30
2	He <sup>3</sup>	0.503	395	1.13
2	He <sup>3</sup>	0.601	395	1.09
2	He <sup>4</sup>	0.700	300	1.11
2	He <sup>4</sup>	0.797	300	1.09
2	He <sup>4</sup>	0.874	300	1.06

also an irreversible change of order 0.1% in the calibration of the internal resistance thermometer.

The pressure effect observed following the initial pressure cycle is summarized in Table I for both liquid He<sup>3</sup> and He<sup>4</sup>. The He<sup>3</sup> data have *not* been corrected for the bulk thermal resistance of liquid He<sup>3</sup>, a fact which may account for part or all of the variation with temperature noted above 0.3°K. Such a correction is not necessary for liquid He<sup>4</sup>, however, and the slight temperature dependence must be attributed to the thermal boundary resistance.

The thermal boundary resistance measurements on liquid He<sup>3</sup> obtained with cell No. 1 are also included in Fig. 3. All the data were arbitrarily normalized by a constant factor such that the 1-psi data for temperatures below 0.1°K would agree with the analogous data from cell No. 2. This was necessary because of the difficulty in extracting the separate contributions of the Epibond 100-A and the copper surfaces to the thermal impedance. The normalization makes possible a comparison of data on the pressure effect from cell No. 1 with data from cell No. 2. It is noted that the pressure effect measured in the two cells is identical within experimental scatter. With a knowledge of the effective boundary resistance of the epoxy resin-He<sup>3</sup> interface in cell No. 1 (see below), it can be said that the pressure effect for a copper-He<sup>3</sup> interface reproduced to within  $\pm 4\%$  and that the pressure effect for an epoxy resin-He<sup>3</sup> interface was the same as that for a copper-He<sup>3</sup> boundary to within  $\pm 20\%$ .

Only data obtained below 0.125°K in cell No. 1 have been included in Fig. 3. At higher temperatures the effect of the heat capacity of the liquid He<sup>3</sup> became increasingly important, as discussed in the section on experimental technique, and was difficult to calculate accurately. The correction was roughly 5% at 0.125°K. It was noted that the  $RT^3$  values at temperatures above 0.12°K definitely fell higher than the data obtained from cell No. 2. The cause of this is not understood. It may indicate improper operation of the inner cylinder, or possibly a temperature dependence for a He<sup>3</sup>-epoxy resin boundary different from that observed for a He<sup>3</sup>-copper boundary at temperatures greater than 0.12°K.

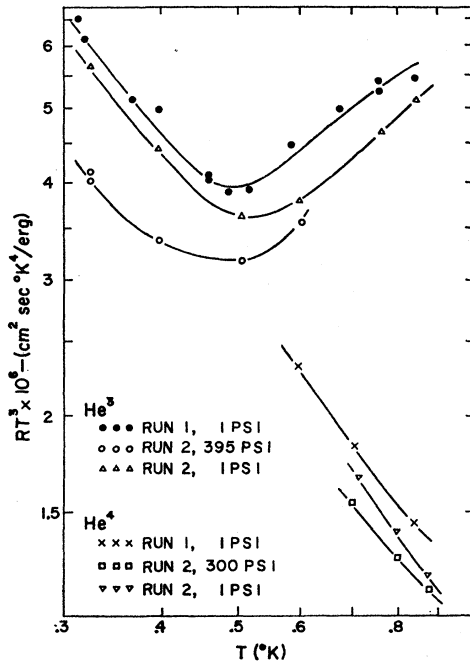


FIG. 5. The product, of the thermal boundary resistivity  $R$  of liquid He<sup>3</sup> or He<sup>4</sup> multiplied by the third power of the temperature, as a function of temperature. The data, which were obtained from cell No. 2, exhibit an irreversible phenomenon in the pressure effect. The He<sup>3</sup> data have not been corrected for the bulk thermal resistance of the liquid.



It will be noted from Fig. 3 that the experimental scatter for cell No. 1 was smaller than that for He<sup>3</sup> measurements with cell No. 2. There was also little scatter in the measurements with liquid He<sup>4</sup> or solid He<sup>3</sup> in cell No. 2. The smaller scatter resulted from the rapidity with which one could measure with liquid He<sup>4</sup> or solid He<sup>3</sup> (short thermal time constant) in cell No. 2, and the greater speed with which it was possible to make measurements with the method used with cell No. 1. A short measuring time is essential to avoid spurious effects due to the uncontrollable change in rf joule heating of the internal resistance thermometer which takes place from time to time.

A rough value of the thermal boundary resistance between liquid He<sup>3</sup> and a wall of Epibond 100-A may be obtained from the data of cells No. 1 and No. 2. The boundary resistance at the surface of the internal copper cylinder of cell No. 1 may be calculated using the data of cell No. 2. This resistance is subtracted from the total measured thermal impedance of cell No. 1. If it is assumed that the Epibond surface in cell No. 1 is smooth, which is not true, the computed He<sup>3</sup>-Epibond 100-A boundary resistivity is

$$R \approx 6 \times 10^{-6} T^{-3} \text{cm}^2 \text{sec}^\circ \text{K}^4 / \text{erg} \quad (T < 0.12^\circ \text{K}). \quad (8)$$

This is larger than the value quoted above for the He<sup>3</sup>-Epibond 100-A surface of the specific heat cells. It should be pointed out, however, that an examination of the inner surface of one of the specific heat cells and a cell constructed for the present work, each having been cut open, revealed a considerable difference in the number of crevices between copper wires, thickness of epoxy, etc.

The thermal boundary resistance between solid He<sup>3</sup> and copper is also shown on Fig. 3. The solid was in the bcc phase and was formed by cooling the cryostat with the pressure on the dead weight tester maintained at 645 psi. The final density of the solid is unknown since there may have been some slippage of the frozen He<sup>3</sup> plug in the fill-tube during solidification. The data may include a contribution from the bulk thermal resistance of the solid He<sup>3</sup>, especially at the highest temperatures. No thermal conductivity measurements on solid He<sup>3</sup> have been made in the region of the bcc phase represented here, and thus a correction is not possible. In none of the graphs of experimental data has any correction been made for spurious thermal resistance *not* in the helium or at the interfaces. Presumably there is at least a thermal resistance proportional to  $T^{-1}$  due to the temperature drop in the copper parts of the cell between the two resistance thermometers. Since the part of  $RT^3$  due to this effect is proportional to  $T^2$ , it becomes less important at low temperatures. Such a contribution to  $RT^3$  would be less than the smallest observed value. Inspection of the He<sup>4</sup> data in Fig. 5 indicates that the resistive effect due to the copper probably has an  $RT^3$  value much less than  $10^{-6} \text{cm}^2 \text{sec}^\circ \text{K}^4 / \text{erg}$  at  $0.9^\circ \text{K}$ .

The dominant features of the data obtained from the five experiments on thermal boundary resistance will now be summarized.

(1) The temperature dependence of the boundary resistance for a liquid He<sup>3</sup>-epoxy resin interface is  $T^{-3}$  from about  $0.1^\circ \text{K}$  down to at least  $0.02^\circ \text{K}$  and, for a liquid He<sup>3</sup>-copper interface, from  $0.11^\circ \text{K}$  down to at least  $0.055^\circ \text{K}$ .

(2) The temperature dependence of the liquid He<sup>3</sup>-copper boundary resistance changes abruptly at  $0.11^\circ \text{K}$  independently of the applied pressure. The temperature dependence of the liquid He<sup>4</sup> boundary resistance, which is not  $T^{-3}$  between  $0.075^\circ \text{K}$  and  $0.85^\circ \text{K}$ , does *not* exhibit an abrupt change as in the case of He<sup>3</sup>. There is also no abrupt change in the slope of  $RT^3$  for He<sup>4</sup> near  $0.6^\circ \text{K}$  indicating that rotons do not play a dominant role, as concluded on a similar basis by Fairbank and Wilks.<sup>30</sup>

(3) The effect of pressure at low temperatures on the thermal boundary resistance between He<sup>3</sup> and copper is to decrease  $RT^3$  by a factor of 1.6 on increasing the pressure from 1 to 395 psi. The pressure effect for a He<sup>3</sup>-epoxy resin interface is the same as that for a He<sup>3</sup>-copper interface within about 20%. At high temperatures very little pressure effect is observed both for He<sup>3</sup> and He<sup>4</sup>.

(4) The thermal boundary resistances for He<sup>3</sup> at 395 psi and He<sup>4</sup> at zero pressure are rather similar from 0.1 to  $0.4^\circ \text{K}$ . The data can not be compared above  $0.4^\circ \text{K}$  since the bulk thermal resistance of liquid He<sup>3</sup> at 395 psi is not known.

#### IV. DISCUSSION

The results from the measurements of the thermal boundary resistance between liquid He<sup>3</sup> or liquid He<sup>4</sup> and copper will be compared with the relevant theoretical investigations and with the work of other experimentalists.

##### A. Comparison with Theory

A substitution into Eq. (1) of the physical constants appropriate to low-pressure liquid He<sup>4</sup> and copper gives the value

$$RT^3 \approx 5 \times 10^{-5} \text{cm}^2 \text{sec}^\circ \text{K}^4 / \text{erg}. \quad (9)$$

There has been some discussion concerning the proper magnitude of the function  $F$  which appears in Eqs. (1) and (2).<sup>31</sup> The value  $F = 1.6$  was used in the calculations in this paper. The disputed variations in value amount to less than 30%, which is small relative to the disagreement between theory and experiment.

Empirically we do not observe a  $T^{-3}$  temperature dependence, but rather an approximate  $T^{-4}$  dependence

<sup>30</sup> H. A. Fairbank and J. Wilks, Proc. Roy. Soc. (London) A231, 545 (1955).

<sup>31</sup> L. C. Challis, in *Proceedings of the Seventh International Conference on Low Temperature Physics* (University of Toronto Press, Toronto, 1961), p. 468.

near 0.6°K which decreases to  $T^{-3.5}$  at 0.09°K. The magnitude of the product,  $RT^3$  increases from  $0.15 \times 10^{-5}$  cm<sup>2</sup>sec°K<sup>4</sup>/erg at 0.8°K to  $1.27 \times 10^{-5}$  cm<sup>2</sup>sec°K<sup>4</sup>/erg at 0.08°K. The pressure effect to be expected from Eq. (1) is

$$(RT^3)_{1\text{psi}} / (RT^3)_{300\text{psi}} \approx 1.8, \quad (10)$$

whereas we observe a ratio of 1.06 at 0.87°K and note a slight increase in this value with a reduction in temperature.

Thus there is no agreement between the predictions of the Khalatnikov theory and our data. It may be noted, however, that with decreasing temperatures our data may be approaching asymptotically the magnitude and the temperature dependence calculated from the Khalatnikov theory.

There are several comparisons that can be made between our data on liquid He<sup>3</sup> and the theories of Bekarevich and Khalatnikov<sup>14</sup> and Khalatnikov.<sup>6</sup> A substitution into Eq. (2) of the physical constants appropriate to low pressure, low temperature, liquid He<sup>3</sup> and copper gives the value

$$RT^3 = 24 \times 10^{-5} \text{ cm}^2 \text{ sec}^2 \text{ K}^4 / \text{erg}. \quad (11)$$

In these calculations we use the values  $F=1.6$  and  $\varphi=1$ . The values for  $a$  and  $b$  have been computed by Salinger<sup>32</sup> and are  $a=0.38$ ,  $b=0.05$ . Empirically we observe a  $T^{-3}$  temperature dependence below 0.11°K, but with a magnitude which is too small by a factor of 12. The  $T^{-3}$  dependence has been observed in experiments which have been performed to as low as 0.01°K and with various surfaces.<sup>5,19,20</sup>

At high temperatures Eq. (1) predicts

$$RT^3 = 12 \times 10^{-5} \text{ cm}^2 \text{ sec}^2 \text{ K}^4 / \text{erg}, \quad (12)$$

for copper and liquid He<sup>3</sup> at low pressure. The magnitude of  $RT^3$  at 0.8°K is sixty times lower than the value given in Eq. (12).

The ratio, for liquid He<sup>3</sup>, or  $RT^3$  at low temperatures to  $RT^3$  at high temperatures may be found by dividing Eq. (2) by Eq. (1):

$$\frac{(RT^3)_{\text{low } T}}{(RT^3)_{\text{high } T}} = \left[ \frac{1}{1 + \frac{b}{a} \frac{\varphi}{F}} \right] \frac{1}{a} \frac{2mc_L}{3p_0} \approx 2. \quad (13)$$

Indeed  $RT^3$  does drop as  $T$  increases, but a well-defined region in which  $RT^3$  is constant at high temperature has not been found. Moreover,  $RT^3$  drops also for He<sup>4</sup> as the temperature increases, and it is quite possible that this effect represents a similarity in behavior of He<sup>3</sup> and He<sup>4</sup> at higher temperatures.

Below 0.1°K an increase in pressure from 1 to 395 psi decreases the measured boundary resistance to either an epoxy resin or a copper wall by a factor of 1.58, whereas the decrease as computed from Eq. (2) should be a factor of about 3. At high temperatures Eq. (1)

TABLE II. A comparison of the wavelengths  $\lambda$  of those thermal phonons in helium which would predominate in the transfer of heat from a copper wall with the wavelength in copper, the estimated asperity of the surface, and the estimated thickness of the dense helium layer at the surface.

	1°K	0.1°K
$\lambda$ (liquid He <sup>3</sup> , low pressure)	31 Å	310 Å
$\lambda$ (liquid He <sup>3</sup> , high pressure)	66 Å	660 Å
$\lambda$ (liquid He <sup>4</sup> , low pressure)	41 Å	410 Å
$\lambda$ (liquid He <sup>4</sup> , high pressure)	59 Å	590 Å
$\lambda$ (solid He <sup>3</sup> )	81 Å	810 Å
$\lambda$ (copper)	850 Å	8500 Å
Thickness, dense-layer		15 Å
Surface asperity	Less than 1000 Å	

predicts that the boundary resistance should decrease by a factor of 3 for such a pressure change. The measured effect is only 1.3 near 0.4°K, where the bulk thermal resistance of the liquid He<sup>3</sup> should make a very small contribution to the ratio.

There is evidence in our data of a "transition" temperature near 0.11°K which is characteristic of liquid He<sup>3</sup>. If this temperature represented the upper limit for the validity of the Fermi liquid theory, or if it were indicative of a transition from a zero sound to an ordinary contribution to the boundary resistance, then one might expect the relation  $\omega\tau=n$  to apply. Here  $\omega$  is the angular frequency of the dominant phonons involved in the heat transfer,  $\tau$  is an appropriate mean lifetime of the He<sup>3</sup> quasiparticle states, and  $n$  is a constant of order unity. Empirically the "transition" temperature is practically independent of pressure. Since  $\omega$  is a function only of temperature, the derived  $\tau$  should also be nearly independent of pressure. The theoretical behavior of  $\tau$  is not simple when the disturbance of the Fermi surface is of order  $kT$ .<sup>15,33</sup> Calculations suggest, however, that  $\tau$  should decrease with increasing pressure and that a transition temperature associated with the condition  $\omega\tau=n$  should therefore decrease upon application of pressure.

Hence we can find no quantitative agreement between theory and our data on liquid He<sup>3</sup> with the exception of the constancy of  $RT^3$  at low temperatures.

The increase of  $RT^3$  with decreasing temperature is similar to what would be expected if van der Waals forces produced a dense layer of helium at the interface.<sup>11</sup> However, the magnitude of this effect as calculated by Challis, Dransfeld, and Wilks is small at low temperatures and does not account for the behavior we observe. In Table II a comparison is made between the estimated thickness of the dense helium layer and the wavelengths of those phonons in the liquid that should be predominant in contributing to the transfer of heat.

Table II also permits a comparison between the wavelength of the phonons and the estimated asperity of the copper surface. The surface of a typical electropolished

<sup>32</sup> G. L. Salinger, Ph.D. thesis, University of Illinois, 1961 (unpublished).

<sup>33</sup> D. Pines (private communication).

piece of copper was examined under a metallurgical microscope and was found to have scattered protuberances of roughly 1000 Å diam. The rest of the field appeared dark. It is probable that the surface asperity is of the order of several hundred angstroms. It is quite likely that the surface roughness influenced the measured thermal boundary resistances at the higher temperatures. As discussed above, the temperature at which the curves of the liquid He<sup>3</sup> boundary resistance abruptly change in slope is nearly independent of the applied pressure. Since the wavelengths of the phonons in the liquid increase by a factor of 2 with pressure, it is unlikely that the abrupt change can be correlated with an interaction of these phonons with the asperity of the surface.

The only well-defined surface used in our experiments has been copper. It is therefore not possible to extract any quantitative information concerning the possible contribution of metallic electrons to the thermal boundary resistance.<sup>12,13</sup> No attempt has been made to compare the boundary resistance of solid He<sup>3</sup> with the theory of Little,<sup>34</sup> other than to observe that no simple temperature dependence is obeyed.

It is of interest to compare the boundary resistance curves for liquid He<sup>3</sup>, Fig. 3, with the curve for He<sup>4</sup>, Fig. 4. The acoustic impedance ( $\rho c$ ) of liquid He<sup>3</sup> at 395 psi is almost the same as the acoustic impedance of liquid He<sup>4</sup> at zero pressure. As noted earlier, the boundary resistance of He<sup>3</sup> at 395 psi is also similar in magnitude to that for He<sup>4</sup> at zero pressure. Thus it appears that at higher temperatures the thermal boundary resistance of He<sup>3</sup> and He<sup>4</sup> are rather similar, though certainly not precisely so, whereas at low temperatures, they are qualitatively different in temperature dependence. That the two are different at low temperatures is not necessarily surprising since it is known that at low enough temperatures the elementary excitations of He<sup>4</sup> are thermal phonons, while those for He<sup>3</sup> are the quasiparticles of the Landau Fermi liquid, perhaps collectively excited by the wall into zero-sound modes.

Another difference between the He<sup>4</sup> and the He<sup>3</sup> is the long mean free path of the thermal phonons in He<sup>4</sup> which should permit multiple reflections in the boundary resistance cell and possibly alter the boundary resistance from that where the phonons from the solid enter a semi-infinite medium.

The magnitude of  $RT^3$ , in the case of liquid He<sup>4</sup>, changes by a factor of 10 over the temperature range of the experiment. Reasonable extrapolation of  $RT^3$  to higher and lower temperatures suggests that the total change in  $RT^3$  is several times the observed factor of 10. Only a small fraction of this very large change could be explained by the increase in effective surface area at high temperatures, where the phonon wavelength becomes small relative to the surface roughness. The high-temperature behavior of  $RT^3$  as a function of tempera-

ture, and the small pressure effect, are indeed qualitatively similar to the predictions of Andreev<sup>13</sup> as to the contribution of metallic electrons to the thermal boundary resistance if, as the temperature decreases, the electron mechanism becomes less powerful. The behavior of  $RT^3$  is also qualitatively similar to the predictions of Challis, Dransfeld, and Wilks<sup>11</sup> as to the effect of a dense surface layer of helium. Whatever the explanation, whether it be a modification of the above effects or a new phenomenon, it must provide a mechanism of heat transfer which at high temperatures is nearly two orders of magnitude more efficient than that provided by the acoustic mismatch theory of Khalatnikov. As evidence of this greater efficiency it may be noted that the Khalatnikov theory predicts, in the case of a copper-He<sup>4</sup> interface, that about  $\frac{1}{2}\%$  of the incident phonon energy flux in the copper is transmitted into the helium. At 0.8°K the actual transmitted energy flux is about 40 times greater than the calculated value corresponding to 20% of the incident phonon energy flux crossing the interface.

## B. Comparison With Other Experiments

The pressure effect measured for liquid He<sup>4</sup> agrees in magnitude and perhaps in temperature dependence with the measurements of Kuang Wey-Yen<sup>9</sup> and Challis, Dransfeld, and Wilks.<sup>11</sup> The magnitude of the boundary resistance of liquid He<sup>4</sup> near 1°K agrees within about a factor of 2 with most previous measurements, although the temperature dependence observed in the present work is generally greater than the temperature dependence reported by others. We are inclined to believe that the irreversible effect observed on increasing the pressure was caused by an irreversible change in the character of the surface, so that, at least above 0.4°K, this effect supports the observation of other workers that the state of the surface is very important.

The magnitude of the boundary resistance of liquid He<sup>3</sup> as reported here is nearly the same as that reported by Lee and Fairbank<sup>2</sup> for the temperature range 0.3 to 0.5°K. Above 0.5°K we find a considerably smaller boundary resistance.

The experiment performed by Keen, Mathews, and Wilks<sup>21</sup> on the acoustic impedance of liquid helium is similar to our measurement of the thermal boundary resistance insofar as both experiments measure the transfer of acoustic energy across a solid-He interface. Indeed the frequencies of the phonons involved in a boundary resistance measurement near 0.02°K are of the same order of magnitude as the frequency used by Keen *et al.*<sup>21</sup> The two experiments were different in that (1) the acoustic measurements involved a quartz-helium interface while the thermal measurements involved a copper-helium interface, (2) the acoustic measurements involved a discrete frequency rather than a continuum of frequencies, and (3) the acoustic measurements seem to involve, at least in the solid, a coherent, longitudinal

<sup>34</sup> W. A. Little, Can. J. Phys. **37**, 334 (1959).

wave incident perpendicular to the surface while the thermal measurements involve, in the solid, randomly incident phonons and, perhaps, electrons as well.

The theories of Khalatnikov<sup>6</sup> and Bekarevich and Khalatnikov<sup>14</sup> may be applied to the He<sup>3</sup> experiment of Keen *et al.* Keeping in mind the conditions outlined in the previous paragraph, one finds that  $\dot{Q}$ , the energy transferred from the solid to the helium, decreases in passing from the high temperature region to the low temperature or Fermi liquid region:

$$\dot{Q}_{\text{high}T}/\dot{Q}_{\text{low}T} = (4c_L/ahN_0)(\pi/\rho_L N_0)^{1/3} \simeq 2, \quad (14)$$

where  $N_0$  is Avogadro's number. Since the energy transfer is proportional to the measured attenuation of the 10<sup>9</sup>-cps sound waves in the quartz, the attenuation should also decrease in passing to lower temperatures. Keen *et al.* however observed an abrupt increase in the attenuation at 0.09°K. It is concluded that the results from the acoustic measurements, as interpreted, are not in agreement with theory.

It is of interest to compare the acoustic measurements with our data without any reference to theory. The similarity of the acoustic impedance and the boundary resistance of high-pressure He<sup>3</sup> and low-pressure He<sup>4</sup> has been noted above. Assuming that above 0.1°K the mechanism responsible for transferring the heat is similar in low-pressure He<sup>4</sup> and high-pressure He<sup>3</sup>, then below this temperature some different and more effective mechanism must exist for transferring heat in the case of He<sup>3</sup>. That is, if  $RT^3$  for liquid He<sup>3</sup> at 0.1°K and above is extrapolated to lower temperatures in Fig. 3, the extrapolation falls at higher values of boundary resistance than are actually measured. In this sense, our data for He<sup>3</sup> indicate a relatively stronger coupling to the liquid at low temperatures and hence are in qualitative agreement with the work of Keen, Mathews, and Wilks.

## V. CONCLUSION

Below about 0.1°K the thermal boundary resistance of liquid He<sup>3</sup> and liquid He<sup>4</sup> in contact with a copper wall are qualitatively different. Below this temperature  $RT^3$  for He<sup>3</sup> is constant, and a reproducible pressure effect is observed. Although  $RT^3$  being constant agrees with theory, neither its magnitude nor the effect of pressure agrees with the theory which is based on a purely acoustic transfer of heat. The boundary resistance between He<sup>4</sup> and copper does not have a  $T^{-3}$  temperature dependence in any part of the temperature range studied. However,  $RT^3$  for He<sup>4</sup> is forty times less than theory at 0.8°K while it is only four times less at 0.07°K. The lack of quantitative agreement with theory prevents us from making any definitive comments about the existence of the phenomenon of zero sound.

In the present experiments the possible influence of the metallic electrons on the boundary resistance could not be isolated; it would be interesting to study this effect with both He<sup>3</sup> and He<sup>4</sup>, especially below 0.1°K in the case of He<sup>3</sup> and probably at much lower temperatures in the case of He<sup>4</sup>. It would be desirable also to extend the measurements on He<sup>4</sup> to lower temperatures to ascertain if the Khalatnikov theory is valid in this limit, and to improve substantially the quality of the surface.

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